Chapter 10

Exercise 10A

1 **a** $y = 5x + 5$

b $y = -5$

c $y = -x - 7$

d $y = 22x - 24$

e $y = 25x + 54$

f $y = -7x + 8$

2 **a** $y = -\frac{1}{2}x + \frac{1}{12}\left(\pi + 6\sqrt{3}\right)$

b $y = \frac{3}{2}x + \frac{1}{6}\left(3\sqrt{3} - \pi\right)$

c $y = 2\sqrt{2}x + 2\sqrt{2}\left(1 - \frac{\pi}{4}\right)$

d $y = -\sqrt{3}x + \frac{1}{12}\left(12 + 7\sqrt{3}\pi\right)$

e $y = \frac{5}{2}x + \frac{5}{2}\left(\sqrt{3} - 1\right)$

f $y = \frac{\sqrt{3}}{4}x - \frac{1}{4}\left(1 + \sqrt{3}\right)$

3 **a** $y = 7x$

b $y = 5x - 3$

c $y = -4x - 5$

d $y = 20x - 72$

e $y = -27x - 46$

f $y = 5x + 8$

4 **a** $y = -\frac{\sqrt{3}}{2}x + \frac{1}{6}\left(3 + \sqrt{3}\pi\right)$

b $y = x + \frac{1}{6}\left(3\sqrt{3} - \pi\right)$

c $y = -4\sqrt{3}x + \frac{4}{3}\left(2\sqrt{3}\pi - 3\right)$

d $y = 6x - \frac{3\pi}{2}$

e $y = 2x - \frac{2}{3}\left(\pi + 3\sqrt{3}\right)$

f $y = -\sqrt{2}x + \frac{1}{8}\left(8\sqrt{2} + 5\sqrt{2}\pi\right)$

5 **a** $y = -6x - 31$

b $y = 5x + 1$

c $y = -6x + \frac{7}{4}$

d $y = 4x + 4\left(\sqrt{3} - 1\right)$

e $y = -2x + \frac{1}{2}\left(\sqrt{3} + \frac{\pi}{6}\right)$

f $y = \sqrt{3}x + \frac{1}{3}\left(3 - \sqrt{3}\pi\right)$

6 **a** $y = 11x - 26$

b $y = 18x + 10$

c $y = -2x + 1$

d $y = 0$

e $y = \frac{1}{4}x + 3$

f $y = \frac{1}{3}x + 12$

7 $y = -8x + 48$

8 **a** $a = -2$

$b = -5$

b $y = 10x - 32$

c $x = \frac{2+\sqrt{31}}{3}$

$x = \frac{2-\sqrt{31}}{3}$

9 **a** $y = 3x - 22$

b $(-4, 74)$

10 $y = 6\sqrt{2}x - \frac{1}{2}\left(4\sqrt{2} + 3\sqrt{2}\pi\right)$

11 $y = 2x - \frac{1}{3}\left(3\sqrt{3} + 2\pi\right)$

12 **a** $y = 10x - 1$

13 **a** $-\frac{2}{49}$

b $y = -\frac{2}{121}x + \frac{17}{121}$

14 $y = 5x - 11$

15 $y = 15x + 14$

16 $y = -2x + \frac{13}{16}$

17 $y = 96x - 272$

18 **a** $x = \frac{3}{4}$

b $y = x + \frac{3}{4}$

19 $y = -\frac{3\sqrt{3}}{8}x + \frac{1}{16}\left(7\sqrt{3}\pi - 2\right)$

$y = -0.6496x + 2.2556$

20 $-\frac{3}{4}x + 9$

21 $y = -\frac{1}{4}x + \frac{19}{4}$

22 a $f\left(g(x)\right) = \sqrt{5 + 8\sin x}$

b i $h'(x) = \frac{4\cos x}{\sqrt{5+8\sin x}}$

ii $\frac{4\cos x}{\sqrt{5+8\sin x}} = \frac{2\sqrt{3}}{3}$

$12\cos x = 2\sqrt{3}\sqrt{5 + 8\sin x}$

$12^2(\cos x)^2 = 12(5 + 8\sin x)$

$12(\cos x)^2 - 8\sin x - 5 = 0$

c $y = \frac{2}{\sqrt{3}}x + \frac{1}{9}\left(27 - \sqrt{3}\pi\right)$

$y = 1.1547x + 2.3954$

23 a $x \neq -3$ & &$x \neq 5$ where && means AND [|| means OR]

b $y = -0.4938x + 3.4074$

24 a $y = -4x + 9$

b $y = 2x$, $B = (3, 6)$

c $(4.2,8.4)$

25 a $y = 2x - 2$

b $\frac{12}{\sqrt{145}}$

c $(5.5,11.25)$

Challenge

1 $-k^3$

2 Half turn because $f(-x) = -f(x)$.
Centre $(0,0)$

3 "a" does not affect any shift in x-axis so ax^3 has same symmetry as x^3.

4 a $g(x) = ax^3 + cx$

$g(-x) = -ax^3 - cx = -g(x)$

so half turn symmetry about origin.

b Half turn about $(0,d)$

5 expand $a\left(x - \frac{b}{3a}\right)^3$ to get

$ax^3 - bx^2 + \frac{xb^2}{3a}\ \frac{-b^3}{27a^2}$

expand $b\left(x - \frac{b}{3a}\right)^2$ to get

$\frac{b^3}{9a^2} - \frac{2b^2x}{3a} + bx^2$

expand $c\left(x - \frac{b}{3a}\right)$ to get

$cx - \frac{bc}{3a}$

adding up above three terms plus additional term d to get

$ax^3 + cx - \frac{b^2x}{3a} + d - \frac{bc}{3a} + \frac{2b^3}{27a^2}$

6 Combining in polynomial of x format:

$ax^3 + \frac{(3ac-b^2)}{3a}x + \frac{27a^2d+2b^3-9abc}{27a^2}$

$p = \frac{3ac-b^2}{3a}$

$q = \frac{27a^2d+2b^3-9abc}{27a^2}$

q is the y-axis origin offset. If subtract q then brings back rotation to centre.

7 $\frac{b}{3a}, q$

Shift by $\frac{b}{3a}$ on x axis and shift by q on y axis.

Exercise 10B

1 a $f'(1) = 5$ so increasing

b $f'(-1) = -4$ so decreasing

c $f'(-1) = -6$ so decreasing

d $f'(-2) = 38$ so increasing

e $f'(-1) = -1$ so decreasing

2 a $x < -1 \,||\, x > 3$ note || means OR

b $-3 < x < -1$

c $x < -\sqrt{2} \,||\, x > \sqrt{2}$

d $x < -2$

e $0 < x < 4$

3 $f'(-1) = 1$ so increasing.

4 $f\left(\frac{1}{3}\right) = \frac{-1}{3}$ so decreasing.

5 a $h'\left(\frac{1}{2}\right) = \frac{3}{2}$ so increasing

b $x < \frac{1}{3}$

6 $g'(2) = -4$ so decreasing

7 $k'(-1) = -48$ so decreasing

8 $f'\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ so increasing

9 $g'\left(\frac{\pi}{6}\right) = 0.5359$ so increasing

10 $p'\left(\frac{\pi}{12}\right) = 0.389$ so increasing

11 $f'(x) = x^2 - 6x + 10$
Completing the square gives
$(x-3)^2 + 1$ – which is always > 0 therefore increasing.

12 $h'(x) = 2x^2 + 4x + 2$
Completing the square gives
$2(x+1)^2$ which is always ≥ 0 ie never negative.

13 **a** $x > \frac{1}{4}$

b $f'(x) = -\frac{2}{(4x-1)^{\frac{3}{2}}}$

$f'\left(\frac{5}{16}\right) = -16$

c Numerator and denominator (because of $\frac{3}{2}$ power) are both positive. External negative sign means negative overall and so decreasing.

14 2

15 $f'\left(\frac{2\pi}{3}\right) = \frac{-9}{4}$.

16 $g'(x) = \frac{x^2+6}{x^2}$ which is always positive, so always increasing.

17 **a** **i** $g(f(x)) = \frac{3}{x^2+4x+9}$

ii $(x+2)^2+5$ Denomenator has no roots; ie is never zero so expression is defined for all x.

b $h'(x) = -\frac{6x+12}{(x^2+4x+9)^2}$

$h'(-3) = \frac{1}{6}$

so increasing

18 A, E

19 **a** $g'(x) = 9 - \frac{1}{(x-1)^2}$

b $g'(x) = \frac{9(x^2-2x+1)-1}{(x-1)^2}$

Denomenator is > 0 so focus on numerator:

$9x^2 - 18x + 8 > 0$

c $x < \frac{2}{3} \,||\, x > \frac{4}{3}$ where || means OR

20 3

21 **a** $p(\sin(ax+b))^2 + p(\cos(ax+b))^2$
$= p(1) = p$
so gradient is zero.

b see (a)

Exercise 10C

1 **a** $\left(-1, \frac{8}{3}\right)$ maximum
$\left(1, \frac{4}{3}\right)$ minimum

b Equation incorrectly formatted.

c $(3,5)$ inflexion.

d $\left(-\frac{2}{3}, \frac{67}{27}\right)$ Maximum
$(2,-7)$ Minimum

e $\left(-1, \frac{15}{2}\right)$ Maximum
$(1.3333, 1.148)$ minimum

f $\left(-\frac{3}{2}, \frac{53}{8}\right)$ Maximum
$\left(\frac{1}{3}, -\frac{25}{54}\right)$ minimum

2 **a** $\left(-\frac{5}{3}, -\frac{121}{27}\right)$ Minimum
$(1,5)$ Maximum

b $\left(\frac{2}{3}, -\frac{14}{27}\right)$ Minimum
$(4,18)$ Maximum

c $(-2,64)$ Maximum
$(2,-64)$ Minimum

d $(0,0)$ Minimum
$\left(\frac{4}{3}, \frac{32}{27}\right)$ Maximum

e $(-3,-18)$ Minimum
$(3,18)$ Maximum

f $(0,0)$ Minimum
$\left(\frac{5}{2}, \frac{125}{24}\right)$ Maximum

3 **a** $(0,0)$ Inflexion
$\left(\frac{3}{2}, -\frac{27}{16}\right)$ Minimum

b $(0,0)$ inflexion
$\left(\frac{9}{2}, -\frac{2187}{16}\right)$ Maximum

c $(-4,-256)$ Minimum
$(0,0)$ Maximum
$(4,-256)$ Minimum

d $\left(-1,\frac{1}{2}\right)$ Maximum

$(0,0)$ Minimum

$\left(1,\frac{1}{2}\right)$ Maximum

e $(-1,-5)$ Inflexion

$(0,-6)$ Minimum

4 **a** $\left(\frac{1}{4},0\right)$

b Inflexion

5 **a** $3\left(x^2-2x-8\right)^2(2x-2)$

b **i, ii**

$(-2,0)$, Inflexion

$(1,-729)$, minimum

$(4,0)$, Inflexion

6 **a** $\dfrac{1}{x(x-6)}$

b $x = 0$

$x = 6$

c **i** $\frac{-(2x-6)}{x^2(x-6)^2}$

ii $\left(3,-\frac{1}{9}\right)$ Maximum

7 There are no stationary points.

8 **a** This can come in many forms, one of which is:

$4\sin x\cos x - 1$

b $(0.2618,-0.1278)$ Minimum

$(1.309,0.557)$ Maximum

9 **a** $3(x-2)^2-5$

b The curve has two stationary points.

10 **a** $y'(x) = 3\left(x^2-5x+8\right)^2(2x-5)$

Only the factor $(2x-5)$ has roots: at $x = \frac{5}{2}$

b $\left(\frac{5}{2},5.360\right)$ minimum

11 **a** **i** $2^3-4(2)^2+2+6=0$

ii $(x-2)(x+1)(x-3)$

b **c**

$x = -1$, min

$x = 2$, max

$x = 3$, min

12 **a** **i** $(-1)^3-4(-1)^2+(-1)+6=0$

ii $x = 2$

$x = 3$

b $(0,72)$, min

$(0.255436, 72.129)$, max

$(11.74456, -5994.1)$, min

13 **a** $f(g(x)) = (x+2)^3-7(x+2)^2$

$= x^3-x^2-16x-20$

b **i** $(-2)^3-(-2)^2-16(-2)-20=0$

ii $(x-5)(x+2)(x+2)$

c $(-2,0)$ Max

$(2.6667,-50.81)$ Min

Exercise 10D

1 **a** roots:

$(0,0)$

$(3,-0)$

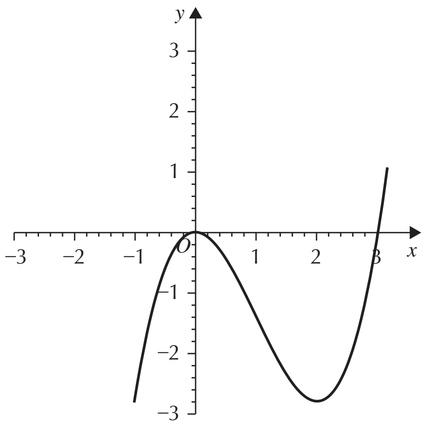
y-axis:

$(0,0)$

Stationary points:

$(0,0)$ Max

$(2,-4)$ Min



b roots:

(−3.464,0)

(0,0)

(3.4641,0)

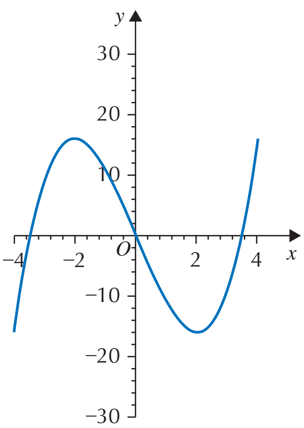
y-axis:

(0,0)

Stationary points:

(−2,16) Max

(2,−16) Min



c roots:

(−1.732,0)

(0,0)

(1.732,0)

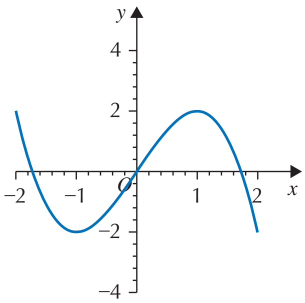
y-axis:

(0,0)

Stationary points:

(−1,−2) Min

(1,2) Max



d roots:

(−2,0)

(1,0)

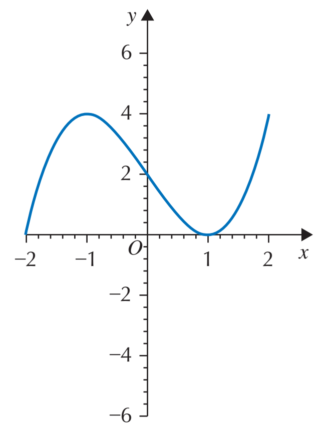
y-axis:

(0,2)

Stationary points:

(−1,4) Max

(1,0) Min



e roots:

(0.5,0)

(2,0)

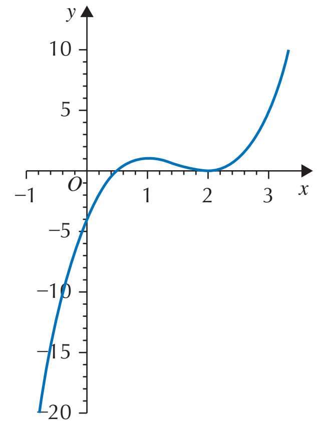
y-axis:

(0,−4)

Stationary points:

(1,1) Max

(2,0) Min



f roots:

(−1,0)

(5,0)

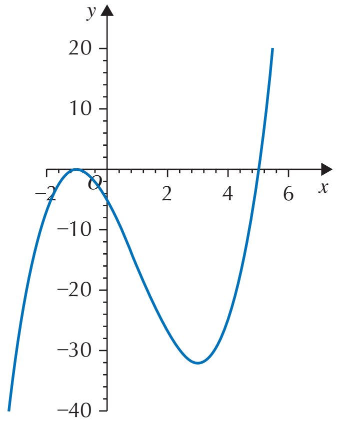
y-axis:

(0,−5)

Stationary points:

(−1,0) Max

(3,−32) Min



g roots:

(0,0)

(2.667,0)

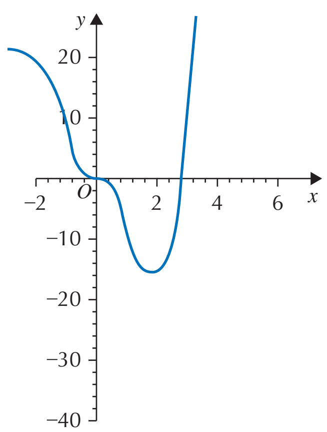
y-axis:

(0,0)

Stationary points:

(0,0) Inflexion

(2,−16) Min



h roots:

(0,0)

(6,0)

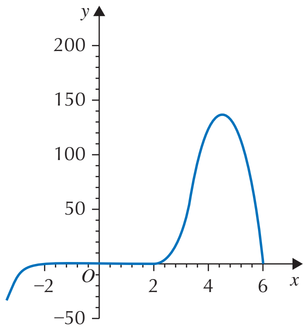
y-axis:

(0,0)

Stationary points:

(0,0) Inflexion

(4.5,136.688) Max



i roots:

(−3,0)

(3,0)

y-axis:

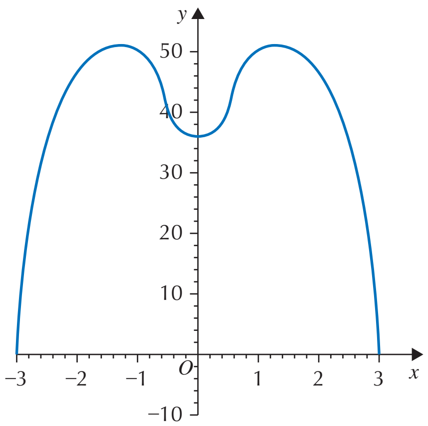
(0,36)

Stationary points:

(−1.581,42.25) Max

(0,36) Min

(1.581,42.25) Max



2 **a** **i** $(1)^3 + 3(1)^2 - 4 = 0$

ii $(x - 1)(x + 2)(x + 2)$

b $(-2,0)$ Max

$(0,-4)$ Min

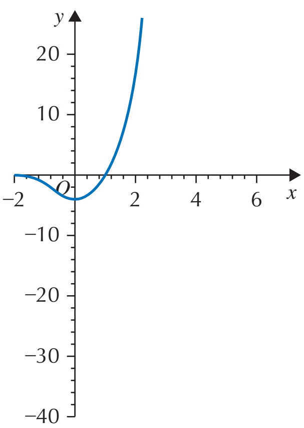
c roots:

$(1,0)$

$(-2,0)$

y-axis:

$(0,-4)$



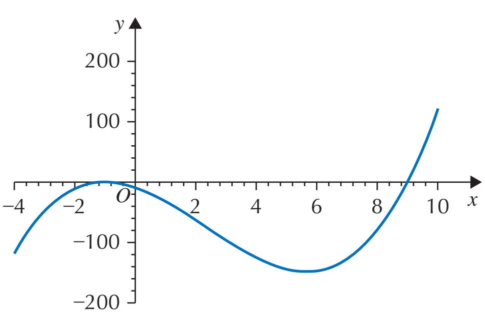
3 **a** **i** $(-1)^3 - 7(-1)^2 - 17(-1) - 9 = 0$

ii $x = 9$

b $(-1,0)$ Max

$(5.667,-148.148)$ Min

c



4 **a** $(x - 2)(x - 2)(x - 6)$

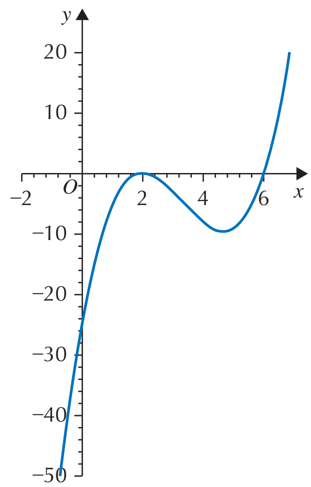
b **i** $(2,0)$

$(6,0)$

ii $(2,0)$ Max

$(4.667,-9.481)$

c



5 **a** **i** $(-2)^3 - (-2)^2 - (-2) + 10 = 0$

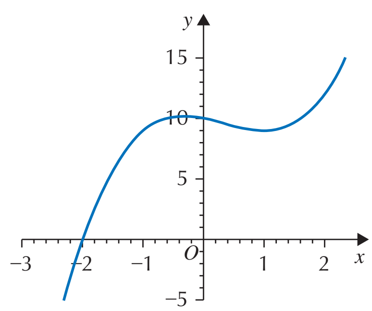
ii $(x + 2)\left(x^2 - 3x + 5\right)$

b **i** $(-0.3333,10.185)$ Max

$(1,9)$ Min

c Only one root – and to left of stationary points.

d



6 **a** **i** $(-2)^4 + 2(-2)^3 - 3(-2)^2$

$-4(-2) + 4 = 0$

ii $(x - 1)(x - 1)(x + 2)(x + 2)$

$= (x - 1)^2(x + 2)^2$

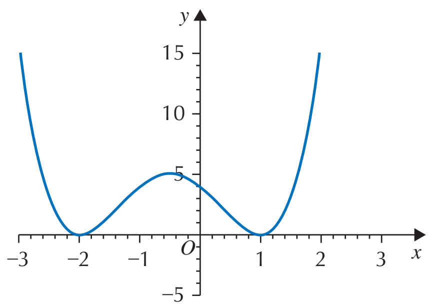
b Product of two squares so cannot be negative.

c **i** $(-2,0)$ Min

$(-0.5,5.0625)$ Max

ii $(1,0)$ Min

d



7 a $k = -4$

b i $(x + 2)(x - 3)^2$

ii There is a root **and** sp at $x = 3$ to y axis must be tangent.

c (–0.3333,18.5185) Max
(3,0) Min

d